Lab 7:

THE MICHELSON INTERFEROMETER (2 Lab Periods)

Objective Calibrate a Michelson interferometer and use it in various applications.

References Hecht, section 9.4; *Universal Interferometer – An Experimental Handbook*

In this set of experiments you will make the following observations and measurements:

- Observe Fizeau and Haidinger fringes for quasi-monochromatic light.
- Observe white-light fringes.
- Calibrate the interferometer's micrometer drive.
- Measure the separation, $\Delta\lambda$, of the sodium D lines (yellow doublet).
- Observe the qualitative difference between the interferograms of light from a Hydrogen gas discharge lamp and light from a HeNe Laser.
- Measure the characteristics of a bandpass filter.

I. Equipment

- Ealing Universal Interferometer, set up as a Michelson interferometer
- Hydrogen discharge lamp
- Red glass or plastic filter; narrow band-pass interference filter
- Tensor lamp (or other suitable white light source)
- He-Ne laser

II. LAB SAFETY:

- **Do not look into the Laser beam. Eye injury and blindness may result.**
- **If you overheat the filters, they may crack. Use the Tensor lamp on its lowest power setting!**

III. Procedure

A. Initial Observations of Interference Fringes

Using either a hydrogen or deuterium lamp as a convenient source of quasi-monochromatic light, adjust the interferometer until you observe interference fringes. Viewing the fringes directly (you do not need a telescope), obtain both circular (Haidinger) and straight (Fizeau) fringes, and be sure that you understand clearly the adjustments that must be made to obtain fringes of each type. You may find it helpful to use a red glass filter here. Why is this so?

Think carefully about the requirements for the observation of white-light fringes, and, using quasi-monochromatic light, adjust the interferometer as close as possible to the configuration for observing white-light fringes. Then substitute an incandescent lamp for the quasi-monochromatic source. You should now be able to find the white-light fringes with only very slight additional adjustments. Make these final adjustments very slowly and delicately. Show your white-light fringes to the instructor and explain the procedure by which you found them. Record the procedure in your laboratory book. Record the reading of the micrometer drive on the movable mirror for the position that gives white-light fringes.

B. Calibration

The micrometer drives the movable mirror through a lever arm. Although the micrometer can be read precisely, this reading cannot be directly translated into mirror motion unless you know the exact reduction factor of this lever arm. Instead, by counting the number of fringes that pass through the field of view for a given micrometer motion the micrometer readings can be calibrated in terms of the wavelength λ of a known spectral line. The passage of one fringe corresponds to a change in optical path length of one wavelength.

For the Michelson interferometer you are using, answer the question: How far must the mirror travel to produce a change of one wavelength in the optical path?

Use the Balmer α line in hydrogen for calibration (the red line with λ = 656.28 nm that derives from the transition between the principal quantum numbers n = 3 and 2). A result accurate to about 1% can be obtained if two partners independently count about 300 fringes. Because this is a tedious process, you may want to count six groups of 50 fringes. It may be helpful if each partner makes two independent counts. Some students have found it easier to carry out this part of the experiment by using a television camera to display the fringes on a monitor.

C. Study of Spectral Lines, Visibility Curves and Interferograms

In general, spectral lines have both width and structure. As you know from an earlier experiment, the yellow D lines in sodium vapor are a doublet with $\Delta\lambda/\lambda_{\text{average}} \sim 10^{-3}$. The structure of the spectrum of a particular source can be described by a *spectral distribution function*, defined as the intensity as a function of wavelength $I(\lambda)$ which can be determined with an optical interferometer. One first measures the *interferogram*: the intensity of the interference pattern as a function of optical path difference between the two mirrors. The spectral distribution function is then proportional to the Fourier transform of the interferogram.

Michelson interferometers are routinely used in infrared spectroscopy. The mirror displacement is under the control of a computer which determines the spectral distribution function from the interferogram using a Fast Fourier Transform (FFT) algorithm.

In distinction, in this part of the lab, we will use a "barefoot approach": If the spectral distribution function is not too complicated, much can be learned from the simpler technique of observing changes in the visibility (or contrast) of the interference fringes. This can be understood from the following simple argument: Suppose the source spectrum consists of two spectral lines of equal intensity whose wavelengths are, respectively, λ' and λ, with $\Delta\lambda = \lambda' - \lambda > 0$ that is small compared with λ. The observed fringe pattern from the doublet will be an incoherent superposition of the individual fringe patterns from the two spectral lines. It will have maximum visibility when the dark fringes for one wavelength essentially coincide with the dark fringes for the other.

Suppose that the m'-th order dark fringe from λ' coincides exactly with the mth order dark fringe from λ at $\theta = 0$ when the mirror separation is d and the optical path difference is $2d$. Assuming that the beam splitter is a dielectric mirror, so that $2d = m\lambda$ gives dark fringes, we can write

$$m'\lambda' = m\lambda = 2d \tag{7.1}$$

Noting that $\lambda < \lambda'$, we see that $m = m' + \mathrm{k}$, where k is a positive integer. We do not, however, know any of the values of m, m', or k. Dark fringes from adjacent orders from λ' will also be in near (although not exact) coincidence with dark fringes from λ, and the fringe pattern will have maximum visibility. Suppose this maximum visibility is found for an optical path difference $2d_1$.

Then using $2d_1/\lambda = m$ and $2d_1/\lambda' = m'$, and combining these expressions with $m = m' + k$, we can write

$$2d_1/\lambda = 2d_1/\lambda' + k \tag{7.2}$$

Because the wavelengths differ slightly, the two sets of fringes will move at slightly different rates as the optical path difference between the two mirrors is varied. If we start from a position of maximum fringe visibility and vary d slowly, we will eventually reach a mirror position for which the dark fringes from one wavelength essentially coincide with the bright fringes from the other wavelength, giving a superposed fringe pattern of almost uniform brightness, with minimum visibility.

Further variation of the optical path difference will again lead to a fringe pattern having maximum visibility. Suppose maximum visibility is next found for an optical path difference $2d_2$. The difference between m and m' will have increased by 1 between adjacent positions of maximum visibility: $m = m' + (k + 1)$. We can then write

$$2d_2/\lambda = 2d_2/\lambda' + (k + 1) \tag{7.3}$$

Combine these two expressions to show that

$$\Delta\lambda = \lambda^2/2(d_2 - d_1) \tag{7.4}$$

if $\Delta\lambda = \lambda' - \lambda$ is small compared with λ. Apply this results to a measurement of the splitting of the Na D doublet.

Questions that you should consider in making these measurements:

(1) Should you measure the change in optical path between adjacent visibility maxima or between adjacent visibility minima? Observe the variation in visibility around a minimum and around a maximum. Which do you think you can locate more precisely?

(2) Do you need to start your measurements from the mirror position that gives zero path difference?

(3) Do you have to count fringes for this measurement?

D. Interferograms of Single Spectral Lines

If a line is Doppler broadened by the distribution of thermal velocities, we expect its profile to be a Gaussian that varies with frequency ν according to

$$I(\nu) = I_0 \exp\left(-\pi\left(\frac{\nu - \nu_0}{\Delta\nu}\right)^2\right) \tag{7.5}$$

with
$$\Delta\nu = \nu_0\sqrt{\frac{2\pi k_B T}{Mc^2}} \tag{7.6}$$

where M is the mass of the emitting atom and T is the absolute temperature of the gas. If the line is *pressure* broadened, the line shape and width are given by different expressions, but for our laboratory sources you may assume that Doppler broadening is the dominant contribution to the line width.

An expression for the visibility curve of a Gaussian spectral line is given in the supplementary notes at the end of this document. Observe the interferogram of the Balmer α line (λ = 656.3 nm) from a hydrogen lamp. A red glass filter may also be helpful here. Can you detect any significant variation in the fringe visibility as the mirror is moved away from zero path difference? If so, estimate the spectral width $\Delta\nu$ of the line; if not, estimate an upper bound for the value of $\Delta\nu$.

Illuminate the interferometer with the light from a He–Ne Laser and observe the fringe pattern projected on a screen. The Laser is considered a highly coherent source with a very narrow spectral width. Is this confirmed by your observations? The theoretical value for the Laser spectral width is given by

$$\Delta\nu_{Laser} \approx \frac{4h\nu\left(\Delta\nu_{thermal}\right)^2}{P} \tag{7.7}$$

where h is Planck's constant, $\Delta\nu_{thermal}$ is the Doppler width given above, and P is the output power which is of the order of one milliwatt for the HeNe Lasers. Estimate the excursion of the interferometer mirror required to observe the spectral width of the Laser emission. Remember that it is a neon atom emitting.

For these measurements, do you need to start from the mirror position that gives zero path difference?

E. Determination of the Band-Pass Characteristics of a Filter

One can determine the band-pass characteristic of a filter by illuminating it with white light and observing the interferogram in the same way that we observed the width of a spectral line. The optics laboratory has a number of reasonably narrow interference filters. Although we may not necessarily know the details of the spectral transmission of these filters, we do have the manufacturer's specifications of the bandwidth and center wavelength for each. If the spectral transmission is Gaussian it may be described in the same manner as the profile of a Doppler-broadened spectral line, and its transmission T may be written as:

$$T(\nu) = T_0 \exp\left(-\pi\left(\frac{\nu-\nu_0}{\Delta\nu}\right)^2\right) \tag{7.8}$$

with a frequency bandwidth $\Delta\nu$ designed by the manufacturer. Note that the filter manufacturer actually specifies λ_0 and $\Delta\lambda$ rather than ν_0 and $\Delta\nu$. Choose a filter from our collection and, assuming that it has Gaussian transmission, observe its interferogram and determine its bandwidth. Your result may differ somewhat from the manufacturer's specifications if the filter transmission is not Gaussian. Compare your measurement with the specifications and try to account for any discrepancies.

APPENDIX: Visibility of a Gaussian Spectral Line

In the experiment on the interferogram of a single spectral line, you need to know the expression for the visibility. We present the result here without derivation. This result is also appropriate for the situation of an interference filter with a Gaussian transmission characteristic.

Consider a single spectral line with a Gaussian spectral intensity distribution:

$$I(\nu) = I_0 \exp\left(-\pi\left(\frac{\nu - \nu_0}{\Delta\nu}\right)^2\right) \tag{7.9}$$

The quantity $\Delta\nu$, which is a measure of the width of the intensity distribution, is assumed to be much less than ν_0, the center frequency. The intensity $I(\nu)$, whose maximum value occurs at $\nu = \nu_0$, is reduced to $1/e \approx 0.3679$ of its maximum value when $\nu = \nu_0 \pm \Delta\nu/\sqrt{\pi}$.

The visibility of the fringes from this spectral line is given by

$$V(\tau) = \exp\left(-\pi(\Delta\nu\tau)^2\right) \tag{7.10}$$

where the *retardation time* τ is defined as $\tau = 2d/c$, and d is the displacement of the movable mirror from its location for zero-path-difference. Note that the retardation time is just the time for light to travel the path difference $2d$. A narrow spectral line (small $\Delta\nu$) requires a large value of τ (and therefore of d) to reduce the visibility significantly. If you recall that the Fourier transform of a Gaussian is another Gaussian it is not surprising that the visibility function is proportional to the Fourier transform of the spectral distribution of the intensity.

According to Michelson (see Hecht section 12.2) the visibility can be written as

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{1 - I_{\min}/I_{\max}}{1 + I_{\min}/I_{\max}} \tag{7.11}$$

where I is the intensity. V can then be determined from a visual estimate of the ratio $I_{\min}/I_{\max}$.